

Short Communication

**SOME PROBLEMS OF FINDING OF EIGENVALUES AND EIGENVECTORS
FOR SH-WAVE PROPAGATION IN TRANSVERSELY
ISOTROPIC PIEZOELECTROMAGNETICS**

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ABSTRACT

This short theoretical work discusses some problems of finding of the suitable eigenvalues and eigenvectors. The eigenvalues and eigenvectors represent the solutions of the coupled equations of motion written in the well-known tensor form. These coupled equations of motion describe shear-horizontal (SH) wave propagation in the transversely isotropic piezoelectromagnetic materials of class $6mm$ when the SH-wave propagation is coupled with both the electrical and magnetic potentials. It is stated that as many as six eigenvalues can be soundly found for the problem. The problem is that some eigenvalues result in the corresponding certain eigenvectors and some eigenvalues allow existence of uncertain eigenvectors that can be chosen by a researcher. This uncertainty allows researchers to choose several certain forms for the uncertain eigenvectors. It is discussed that the author of this report has used the certain forms for the uncertain eigenvectors that are naturally coupled with the certain eigenvectors. However, some researchers suggest to use the following forms for the uncertain eigenvectors: $(0,1,0)$ and $(0,0,1)$. It is stated that the simplest and perhaps convenient eigenvectors in the forms of $(0,1,0)$ and $(0,0,1)$ are actually unsuitable because they are independent from the certain eigenvectors and the CMEMC coupling mechanisms. It is very important to use suitable eigenvectors because different forms of them can result in different final expressions for the velocities of the SH-waves. The SH-wave velocity is a very important wave characteristic and evaluation of its value can help for creation and optimization of novel technical devices based on surface, interfacial, and plate SH-waves.

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INTRODUCTION

This theoretical work discusses the problems of finding of suitable forms of the eigenvalues and eigenvectors. The found forms of the eigenvalues and the corresponding eigenvectors represent the solutions of the coupled equations of motion written in the well-known tensor form and can result in final expressions for the suitable propagation velocities of the shear-horizontal (SH) acoustic waves coupled with both the electrical and magnetic potentials. This difficulty touches the propagation problems of the surface (Zakharenko, 2010), interfacial (Zakharenko, 2012a), and plate (Zakharenko, 2012b) SH-waves in the transversely isotropic piezoelectromagnetics of class $6mm$.

The piezoelectromagnetic (composite) materials, also known as the magnetoelastoelectric media, are eminent as smart materials due to the fact that the electrical subsystem of the materials can interact with the magnetic

subsystem via the mechanical subsystem, and vice versa. Therefore, it is vital to be familiar with the wave characteristics of such (composite) materials due to possible constitution of new technical devices with a high level of integration. It is understandable that this knowledge can help for further miniaturization of various technical devices and be used for the nondestructive testing and evaluation of piezoelectromagnetic (composite) materials. There is currently much review work on the magnetolectric effect and the piezoelectromagnetics possessing this effect together with the other effects such as the piezoelectric and piezomagnetic effects. It is thought that the most complete list of review works on the subject is given in a first review paper by Zakharenko (2013a) concerning the SH-wave propagation problems in such smart materials.

The discussions highlighted in this short report are based on the results obtained in theoretical works (Zakharenko, 2010; Zakharenko, 2012a; Zakharenko, 2012b) that use certainly found eigenvalues and eigenvectors. However, some researchers believe that the natural eigenvalues and

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the corresponding eigenvectors exploited in books (Zakharenko, 2010; Zakharenko, 2012a; Zakharenko, 2012b) for the transversely isotropic case cannot be the single possibility. They offer the other eigenvectors that actually represent an artificial case. This is also discussed below. Also, it is necessary to mention recent theoretical works (Wang *et al.*, 2007; Liu *et al.*, 2007; Wei *et al.*, 2009; Melkumyan, 2007) that do not provide any eigenvalues and eigenvectors. The main expressions and discussions are given below in the following section.

Problems of finding of eigenvalues and eigenvectors

It is central to know the propagation directions of the shear-horizontal (SH) elastic waves when they can be coupled with both the electrical and magnetic potentials. For the transversely isotropic piezoelectromagnetic material of class $6mm$, the suitable propagation direction is given in the review paper written by Gulyaev (1998) and the coordinate system is shown in review paper by Zakharenko (2013a). This propagation direction for the $6mm$ materials is different from the suitable propagation direction for the cubic piezoelectromagnetics (Zakharenko, 2013a; Zakharenko, 2011). Following this review paper, it is possible to state that the SH-wave propagation direction must be managed along the free surface of the piezoelectromagnetics when both the surface normal and the propagation direction are perpendicular to the sixfold symmetry axis of the $6mm$ material. In this configuration, the SH-waves are polarized along the sixfold symmetry axis. This is the case of pure waves where the pure SH-waves can be separately studied (Lardat *et al.*, 1971; Dieulesaint and Royer, 1980). To study the SH-wave propagation, the quasi-static approximation (Dieulesaint and Royer, 1980; Auld, 1990) must be used when the SH-waves are coupled with both the electrical and magnetic potentials.

In the case of the pure SH-wave propagation, the corresponding tensor form of the equations of motion for the piezoelectromagnetic ($6mm$) medium can be written (Zakharenko, 2010; Zakharenko, 2012a; Zakharenko, 2012b; Zakharenko, 2013a). Also, it is essential to mention the following nonzero material parameters (Zakharenko, 2010; Zakharenko, 2012a; Zakharenko, 2012b; Zakharenko, 2013a; Nye, 1989; Newnham, 2005): the elastic stiffness constant C , piezoelectric constant e , piezomagnetic coefficient h , dielectric permittivity coefficient ε , magnetic permeability coefficient μ , and electromagnetic constant α . These material constants are defined as follows: $C = C_{44} = C_{66}$, $e = e_{16} = e_{34}$, $h = h_{16} = h_{34}$, $\varepsilon = \varepsilon_{11} = \varepsilon_{33}$, $\mu = \mu_{11} = \mu_{33}$, and $\alpha = \alpha_{11} = \alpha_{33}$ (Zakharenko, 2010; Zakharenko, 2012^a; Zakharenko, 2012^b; Zakharenko, 2013^a). Solving the corresponding equations of motion written in the tensor form (Zakharenko, 2010; Zakharenko, 2012a; Zakharenko, 2012b), it is possible to obtain all the eigenvalues and the corresponding eigenvectors. To discuss the problem of

finding of suitable eigenvalues and eigenvectors is the main purpose for this short report.

When the SH-wave propagation is coupled with both the electrical (φ) and magnetic (ψ) potentials, the corresponding tensor form of the coupled equations of motion can be expressed by three homogeneous equations written in the following matrix form (Zakharenko, 2010; Zakharenko, 2012a; Zakharenko, 2012b):

$$\begin{pmatrix} C[m - (V_{ph}/V_{t4})^2] & em & hm \\ em & -\varepsilon m & -\alpha m \\ hm & -\alpha m & -\mu m \end{pmatrix} \begin{pmatrix} U^0 \\ \varphi^0 \\ \psi^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

where $m = 1 + n_3^2$ and $(U^0, \varphi^0, \psi^0) = (U_2^0, U_4^0, U_5^0)$ are unknown and must be found. n_3 represents the eigenvalues and U^0 , φ^0 , and ψ^0 are the eigenvector components. In equations (1), ρ and V_{ph} are the mass density of the piezoelectromagnetic material and the phase velocity, respectively. The phase velocity V_{ph} is defined by the following relation: $V_{ph} = \omega/k$, where ω is the angular frequency and k is the wavenumber in the propagation direction of the SH-waves. Also, V_{t4} stands for the speed of the shear-horizontal bulk acoustic wave (SH-BAW) uncoupled with both the electrical and magnetic potentials, $V_{t4} = \sqrt{C/\rho}$.

All the suitable eigenvalues n_3 can be determined when the determinant of the coefficient matrix in equations (1) vanishes. Expanding this matrix determinant, the following secular equation representing a polynomial must be obtained:

$$m \times m \times \left[(1 + K_{em}^2)m - (V_{ph}/V_{t4})^2 \right] = 0.$$

This polynomial is already written in a convenient form of three factors because to find the polynomial roots means to write a polynomial as the suitable factors. So, the first, second, and third factors of the polynomial written above reveal the following eigenvalues:

$$n_3^{(1)} = -n_3^{(2)} = -j \quad (2)$$

$$n_3^{(3)} = -n_3^{(4)} = -j \quad (3)$$

$$n_3^{(5)} = -n_3^{(6)} = -j \sqrt{1 - (V_{ph}/V_{tem})^2} \quad (4)$$

In definition (4), the velocity denoted by V_{tem} represents the speed of the SH-BAW coupled with both the electrical and magnetic potentials. It is defined by the following expression:

$$V_{tem} = V_{t4} (1 + K_{em}^2)^{1/2} \tag{5}$$

In expression (5), K_{em}^2 stands for the coefficient of the magnetoelectromechanical coupling (CMEMC). It can be calculated with the following formula:

$$K_{em}^2 = \frac{\mu e^2 + \varepsilon h^2 - 2\alpha e h}{C(\varepsilon \mu - \alpha^2)} = \frac{e(e\mu - h\alpha) - h(e\alpha - h\varepsilon)}{C(\varepsilon \mu - \alpha^2)} \tag{6}$$

It is understandable in equality (6) that the CMEMC can be represented as the material parameter depending on the following three different coupling mechanisms (Zakharenko, 2013b; Zakharenko, 2013c):

$$e\mu - h\alpha, e\alpha - h\varepsilon, \varepsilon\mu - \alpha^2 \tag{7}$$

With the found eigenvalues defined by expressions (2), (3) and (4), it is necessary to determine the corresponding eigenvectors. Using coupled equations (1), it is also possible to determine the eigenvector explicit forms such as (U^0, φ^0, ψ^0) for all the suitable eigenvalues n_3 . It is natural to use the first equation in equations' set (1) to demonstrate the dependence of the eigenvector component U^0 on both the components φ^0 and ψ^0 . Thus, this dependence reads

$$U^0 = -\frac{em}{A} \varphi^0 - \frac{hm}{A} \psi^0 \tag{8}$$

Exploiting equation (8) for the second and third equations in equations' set (1), one can exclude the eigenvector component U^0 to deal with only two equations in two unknowns such as φ^0 and ψ^0 . Consequently, these two equations are composed as follows:

$$\left(\frac{me^2}{A} + \varepsilon\right) \varphi^0 + \left(\frac{meh}{A} + \alpha\right) \psi^0 = 0 \tag{9}$$

$$\left(\frac{meh}{A} + \alpha\right) \varphi^0 + \left(\frac{mh^2}{A} + \mu\right) \psi^0 = 0 \tag{10}$$

where

$$A = C \left[m - \left(V_{ph} / V_{t4} \right)^2 \right] \tag{11}$$

It is now necessary to state that all the suitable eigenvector components can be obtained by means of the use of each of the found eigenvalues defined by equations from (2) to (4). As soon as each of the eigenvalues is used for equations from (8) to (10), the corresponding eigenvector components are determined. Indeed, each eigenvalue possesses its own certain set of the eigenvector

components and it is natural to use the obtained equations from (8) to (10) to define the components because these equations follow from the coupled equations of motion written in matrix form (1). However, equations (9) and (10) allow each eigenvalue to naturally have two different sets of the eigenvector components. They are discussed below as cases (i) and (ii).

Case (i)

In this case, equations (8) and (9) are used. Accounting the fact that $m = 1 + n_3^2 = 0$ for eigenvalues (2) and (3), it is possible to have the following eigenvector components such as $U^0, \varphi^0,$ and ψ^0 :

$$(U^{0(1)}, \varphi^{0(1)}, \psi^{0(1)}) = (U^{0(3)}, \varphi^{0(3)}, \psi^{0(3)}) = (0, \alpha, -\varepsilon) \tag{12}$$

One can find that the found eigenvector components defined by expression (12) satisfy three homogeneous equations (1). This is true because $m = 0$ leads to all zero components for the second and third columns of the coefficient matrix in equations (1). Therefore, a wavevector of the following form is valid for this case: $(0, x, y)$ where $x \neq 0$ and $y \neq 0$. This uncertainty is naturally resolved by the use of eigenvector (12) coupled with the certain eigenvector corresponding to eigenvalue (4). Employing expressions (8) and (9) for eigenvalue (4) with $m \neq 0$, the corresponding eigenvector components $(U^0, \varphi^0,$ and $\psi^0)$ can be written as follows:

$$(U^{0(5)}, \varphi^{0(5)}, \psi^{0(5)}) = \left(\frac{e\alpha - h\varepsilon}{CK_{em}^2}, \frac{eh}{CK_{em}^2} + \alpha, \frac{e^2}{CK_{em}^2} - \varepsilon \right) \tag{13}$$

$$= \frac{1}{K_{em}^2} ((e\alpha - h\varepsilon)/C, \alpha(K_{em}^2 - K_{\alpha}^2) - \varepsilon(K_{em}^2 - K_e^2))$$

It is very important to state that the found eigenvector components $\varphi^{0(5)}$ and $\psi^{0(5)}$ from expression (13) satisfy both equations (9) and (10).

The eigenvector components such as φ^0 and ψ^0 of eigenvectors (12) and (13) are naturally coupled as follows:

$$e\varphi^{0(1)} + h\psi^{0(1)} = e\varphi^{0(3)} + h\psi^{0(3)} = e\varphi^{0(5)} + h\psi^{0(5)} = e\alpha - h\varepsilon \tag{14}$$

It is clearly seen in equalities (14) that the second coupling mechanism of three CMEMC mechanisms (7) couples the eigenvector components. This reveals the physical sense for the found eigenvectors (12) and (13).

In expression (13), the coefficient of the electromechanical coupling (CEMC) is denoted by K_e^2

and the other parameter denoted by K_α^2 couples only the terms with the electromagnetic constant α in CMEMC (6). They are respectively defined as follows:

$$K_e^2 = \frac{e^2}{C\epsilon}, K_\alpha^2 = \frac{eh}{C\alpha} = \frac{\alpha eh}{C\alpha^2} \tag{15}$$

Case (ii)

In this second natural case, it is natural to utilize equations (8) and (10) to obtain the eigenvector components such as $U^0, \varphi^0,$ and ψ^0 . Using $m = 1 + n_3^2 = 0,$ it is possible to have the following eigenvector components for eigenvalues (2) and (3):

$$(U^{0(1)}, \varphi^{0(1)}, \psi^{0(1)}) = (U^{0(3)}, \varphi^{0(3)}, \psi^{0(3)}) = (0, \mu, -\alpha) \tag{16}$$

For eigenvalue (4) with $m \neq 0,$ the corresponding eigenvector components are found as follows:

$$(U^{0(5)}, \varphi^{0(5)}, \psi^{0(5)}) = \left(\frac{e\mu - h\alpha}{CK_{em}^2}, -\frac{h^2}{CK_{em}^2} + \mu, \frac{eh}{CK_{em}^2} - \alpha \right) \\ = \frac{1}{K_{em}^2} ((e\mu - h\alpha)/C, \mu(K_{em}^2 - K_m^2), -\alpha(K_{em}^2 - K_\alpha^2)) \tag{17}$$

where

$$\epsilon\phi_{0(1)} + \nu\hbar_{0(1)} = \epsilon\phi_{0(3)} + \nu\hbar_{0(3)} = \epsilon\phi_{0(2)} + \nu\hbar_{0(2)} = \epsilon\tau - \nu\alpha \tag{18}$$

Equalities (18) also disclose the physical sense because eigenvector components φ^0 and ψ^0 in eigenvectors (16) and (17) are coupled via the first mechanism of three coupling mechanisms (7) of CMEMC (6).

It is vital to state here that the found eigenvector components $\varphi^{0(5)}$ and $\psi^{0(5)}$ from expression (17) also satisfy both equations (9) and (10). In expression (17), the non-dimensional parameter K_m^2 called the coefficient of the magnetomechanical coupling (CMMC) is defined by

$$K_m^2 = \frac{h^2}{C\mu} \tag{19}$$

Case (iii)

This case relates to the other mathematically possible forms for the eigenvectors that can be even mathematically more convenient but have no any physical

sense. These eigenvectors are possible because of the uncertainty for the case of $m = 0$ for eigenvalues (2) and (3). Therefore, $m = 0$ leads to all the zero components in the second and third columns of the coefficient matrix in equations (1). As a result, some researchers suggest the use of the following form of the eigenvectors for eigenvalues (2) and (3): $(0, x, y)$ where either $x = 1$ or $y = 1,$ or $x = y = 1$ occurs. The latter representing more complicated case will be not discussed in this report below. In addition to certain eigenvectors (13) and (17) for eigenvalue (4), some researchers believe that possible convenient eigenvector forms can be chosen for the eigenvalues defined by expressions (2) and (3) as follows:

$$(0,1,0), (0,1,0) \tag{20}$$

$$(0,0,1), (0,0,1) \tag{21}$$

$$(0,1,0), (0,0,1) \tag{22}$$

$$(0,0,1), (0,1,0) \tag{23}$$

It is clearly seen that none of the artificial eigenvectors defined by expressions from (20) to (24) has any connection to certain eigenvectors (13) and (17). It is thought that eigenvectors (20) and (21) of all the artificial eigenvectors are more referable compared with the other two eigenvectors defined by expressions (22) and (23) because it is natural to deal with the same eigenvectors in the case of identical eigenvalues (2) and (3). Indeed, eigenvectors (22) and (23) were composed by mixing eigenvectors (20) and (21). However, some researchers would like to exploit mixed eigenvectors (22) and (23) because they do not lead to uncertainty for the phase velocity in comparison with eigenvectors (20) and (21). It is necessary to state that eigenvectors (12) and (16) also leads to the uncertainty for the phase velocity that is readily resolved because the corresponding mechanism of three coupling mechanisms (6) of CMEMC (7) works. The artificial eigenvectors defined by expressions from (20) to (23) have no any connection with CMEMC coupling mechanisms (6) and certain eigenvectors (13) and (17). As a result, they can actually lead to fake results for the phase velocity.

CONCLUSION

This report has briefly discussed the problem of finding of the eigenvalues and the corresponding suitable eigenvectors. Due to the discussed uncertainty for eigenvalues (2) and (3), the corresponding eigenvectors can be chosen in several different ways. Three cases were discussed, of which the first and second are natural because they are based on equations from (8) to (10) which were received from coupled equations of motion written in matrix form (1). On the other hand, it was also discussed the other eigenvectors given by expressions from (20) to (23) that have mathematically convenient forms but have no physical sense because they have no any connection with the certain eigenvectors defined by

expressions (13) and (17) and cannot demonstrate any connection with one of CMEMC coupling mechanisms (7).

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